THEORY OF RADIATIVE B DECAYS

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Theory of charmless radiative B decays is reviewed. Existence of uncontrolled non-perturbative effects in the inclusive rate at $\mathcal{O}(\alpha_s)$ is reminded.

Charmless radiative $\bar{B}$ decays are the decays $\bar{B} \to X_{\text{no charm}}\gamma$, where $X_{\text{no charm}}$ is either a particular hadronic state for an exclusive decay, or just any charmless hadronic state in the inclusive case. Such decays are generated by tree-level $b \to u$ transitions with photon radiation, loop-mediated $b \to d$ transitions, and loop-mediated $b \to s$ transitions. Examples of diagrams contributing to each of these three types of transitions are presented in Fig. 1. The loop-mediated transitions are known to be very sensitive to new physics, e.g. to existence of SUSY particles with masses below 1 TeV. In the Standard

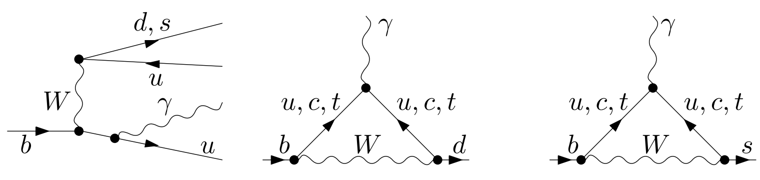


Figure 1. Examples of Feynman diagrams contributing to $\bar{B} \to X_{\text{no charm}}\gamma$.

Model, the $b \to u$ and $b \to d$ transitions are CKM-suppressed with respect to the $b \to s$ ones. The relative suppression factors are $|V_{ub}/V_{ts}|^2 \simeq 1\%$ and $|V_{td}/V_{ts}|^2 \in [2.5\%, 5\%]$, respectively. Therefore, to a good approximation, the loop-mediated $b \to s$ transitions saturate the inclusive $\bar{B} \to X_{\text{no charm}}\gamma$ branching ratio. This branching ratio is measured by CLEO[1] and ALEPH[2] at the level of around 3×10^{-4}, after the contribution from intermediate ψ states is subtracted.

$$\sum_{X_s} BR[\bar{B} \to X_s\gamma]_{\text{loop mediated}} \simeq \sum_{X_{\text{no charm}}} BR[\bar{B} \to X_{\text{no charm}}\gamma]$$
$$\simeq (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4} \qquad (\textbf{CLEO})^1$$
$$+ BR[\bar{B} \to X^{(1)}_{\text{no charm}}\psi] \times BR[\psi \to X^{(2)}_{\text{no charm}}\gamma]. \quad (1)$$

The latter term in the above equation is the very contribution from intermediate ψ. A lower bound on its numerical size can be found by summing up the exclusive branching ratios of charmless radiative ψ decays listed in the Particle Data Book[3] (which give together around 4%) and multiplying them by $BR[\bar{B} \to X\psi]$ that is close to 1%. It follows that the intermediate ψ contribution to $BR[\bar{B} \to X_s\gamma]$ is not smaller than 4×10^{-4}, i.e. it is larger than the remainder of this branching ratio.

The CLEO result is interpreted here as the one with subtracted intermediate ψ contribution, even though no subtraction has been actually made in the measurement performed for high-energy photons.[4] However, the extrapolation to lower photon energies has been done with use of a theoretical model that did not include intermediate ψ. Photons originating from the intermediate ψ are expected to be softer than most of the other photons in $\bar{B} \to X_s\gamma$. A quantitative estimate of their softness (missing at present) is necessary to completely clarify this point.

The intermediate ψ contribution had to be included in Eq. (1), because each $X_{\text{no charm}}$ is assumed to be a QCD-eigenstate, while ψ is not stable in QCD. In other words, the diagrams in Fig. 1, when dressed by an appropriate number of gluons, give a contribution to the intermediate ψ channel as well. Thus, a separation of the intermediate ψ contribution, which may be straightforward on the experimental side, has to be thought about on the theoretical side, too.[5,15] We shall come back to this point later.

Most of this talk will be devoted to the loop-mediated $\bar{B} \to X_s\gamma$ decay that is dominant in $\bar{B} \to X_{\text{no charm}}\gamma$. In order to make a theoretical prediction, we first need to calculate the perturbative b-quark decay amplitudes to partonic states $X_s^{(p)}$ and the photon. Later, the perturbative amplitudes will enter directly into the expressions for hadronic branching ratios. A lot of effort has been devoted in the recent years to calculating the b-quark decay amplitudes with better than 10% accuracy. Single gluon corrections (Fig. 2b) to the one-loop $b \to s\gamma$ diagrams (Fig. 2a) increase the predicted amplitude by around 50%, and the branching ratio by around 100%. This effect is so large because the logarithm $\ln \frac{M_W^2}{m_b^2}$ is big and because the one-loop result is accidentally quite small - it gives only about $\frac{1}{5}$ of what is naively expected. In order to achieve better than 10% accuracy, one needs to include the NLO QCD corrections (Figs. 2c and 2d), i.e. non-logarithmic parts of two-loop diagrams[6,7] and logarithmic parts of three-loop diagrams.[8] The NLO corrections further increase the predicted branching ratio by around 20%. Both the LO and the NLO calculations include resummation of large logarithms $\ln \frac{M_W^2}{m_b^2}$ from all orders of the perturbation series.

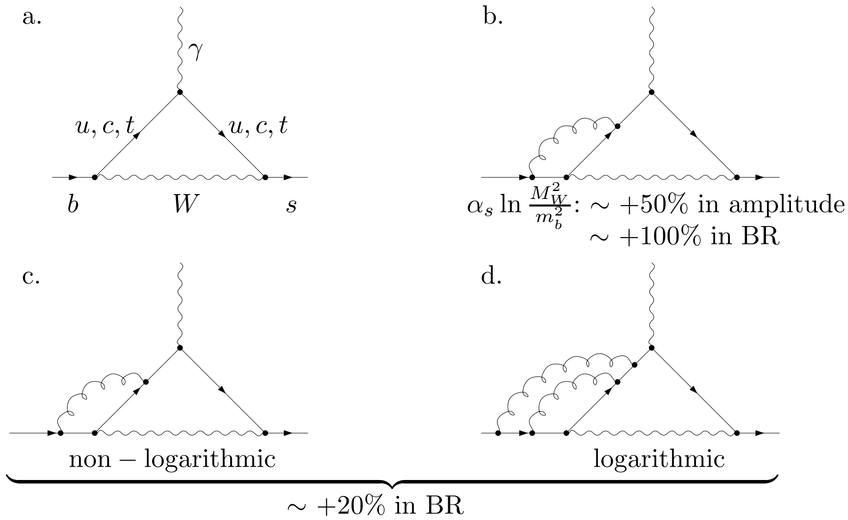


Figure 2. Examples of Feynman diagrams contributing to $b \to s\gamma$ at various orders in the renormalization-group-improved perturbation theory.

Resummation of large logarithms as well as further calculation of hadronic decay rates is most conveniently performed in the framework of an effective theory obtained from the SM by decoupling the heavy electroweak bosons and the top quark. The Lagrangian of the effective theory reads

$$\mathcal{L} = \mathcal{L}_{QCD\times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i, \tag{2}$$

where the first term is just the QCD×QED Lagrangian for the light quarks, and the second term contains flavour-changing local interactions O_i of either 4 quarks or 2 quarks and gauge bosons.

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_{\underline{i}} b), & i = 1,2, & |C_i(\mu_b)| \sim 1, \\ (\bar{s}\Gamma_i b)\sum_q (\bar{q}\Gamma'_{\underline{i}} q), & i = 3,4,5,6, & |C_i(\mu_b)| < 0.07, \\ \frac{em_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, & |C_7(\mu_b)| \sim 0.3, \\ \frac{gm_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, & |C_8(\mu_b)| \sim 0.15. \end{cases} \tag{3}$$

The symbols Γ_i and Γ'_i in O_1,..., O_6 stand for various products of the Dirac and colour matrices. The $\overline{MS}$-renormalized couplings C_i at the scale $\mu_b \sim m_b$ are known nowadays up to (and including) the following terms in their

perturbative expansion:[8,9]

$$C_i(\mu_b) = C_i^{(0)}(\mu_b) + \frac{\alpha_{em}}{\alpha_s(\mu_b)} C_i^{(0)em}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} C_i^{(1)}(\mu_b) + ... \tag{4}$$

Once the Wilson coefficients $C_i(\mu_b)$ are known, the b-quark decay amplitudes are given by Feynman diagrams with single insertions of the flavour-changing interactions, i.e. by matrix elements of the operators O_i between the appropriate partonic states.

For the exclusive decay $\bar{B} \to \bar{K}^*\gamma$, one needs to know matrix elements of those operators between the relevant hadronic states: $\langle \bar{K}^*\gamma | O_i | \bar{B} \rangle$. There have been many attempts to calculate these matrix elements using quark models, QCD sum rules, lattice and heavy quark symmetries (see e.g.[10]). The history of published predictions is briefly summarized in Fig. 3. The (blue) thin bars and dots are the theoretical predictions, while the (red) thick bars denote the CLEO measurements. Many recent theoretical papers on $\bar{B} \to \bar{K}^*\gamma$ are not included in the plot because only form-factors are discussed there, and no explicit number for the decay rate is given. A general conclusion one

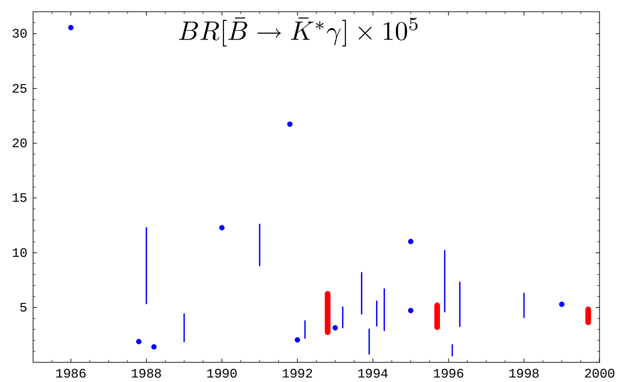


Figure 3. Brief history of published predictions and experimental results for $BR[\bar{B} \to \bar{K}^*\gamma]$.

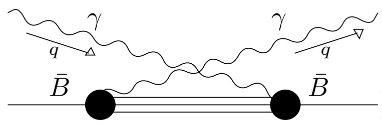
can derive from the $\bar{B} \to \bar{K}^*\gamma$ studies is that they can help us in understanding non-perturbative QCD, but $BR[\bar{B} \to \bar{K}^*\gamma]$ is not a good place to look for new physics, given the present experimental and theoretical results for the inclusive mode $\bar{B} \to X_s\gamma$. However, observation of a sizable CP-asymmetry in the exclusive mode would be a clear signal of new physics.[11]

For the inclusive decay $\bar{B} \to X_s\gamma$, the theoretical prediction for the branching ratio can be made more precise by using the Operator Product Expansion (OPE) within the Heavy Quark Effective Theory (HQET).[12] We

need to calculate

$$\sum_{X_s} \left| C_7 \langle X_s \gamma | O_7 | \bar{B} \rangle + C_2 \langle X_s \gamma | O_2 | \bar{B} \rangle + ... \right|^2 , \tag{5}$$

where dots stand for matrix elements of other operators that are numerically less important. Let us first look at the "77" term, i.e. the term proportional to $|C_7|^2$ in Eq. (5). This term dominates in the perturbative calculation of the b-quark decay. In full analogy to the semileptonic B-meson decay, we relate it via optical theorem to the imaginary part of the elastic forward scattering amplitude

$Im\{$  $\} \equiv Im\ A$ (6)

In this amplitude, we can perform OPE when the photon energies E_γ in the $\bar{B}$-meson rest frame are far from the endpoint, i.e. when $|m_B - 2E_\gamma| >> \Lambda_{QCD}$. Most of the photons in $\bar{B} \to X_s \gamma$ have energies close to the endpoint $E_\gamma^{\max} \simeq \frac{1}{2} m_b$, so they do not satisfy this requirement. Thus, at this first step, OPE gives us only the tail of the photon spectrum.

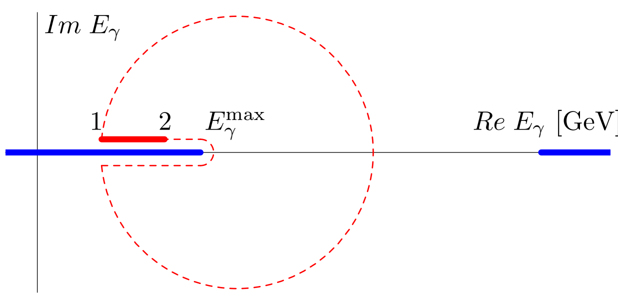


Figure 4. Physical cuts in $A(E_\gamma)$ (thick lines on the real axis) and the relevant contour.

Fortunately, we know analytic properties of the amplitude A when E_γ is formally treated as complex. We know that the discontinuity of A on the real axis is equal to ImA. Thus, if we want to know the integral of ImA from, say, 1 GeV to the endpoint, we can find it by performing the integral of A around the big circle in Fig. 4, where the condition for OPE is always fulfilled.

$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma\ E_\gamma^n\ Im\ A(E_\gamma) \sim \oint_{\text{big circle}} dE_\gamma\ E_\gamma^n\ A(E_\gamma), \qquad n = 0, 1, 2, ... \tag{7}$$

We should better not go with E_γ much below 1 GeV. There is no problem in doing so for the "77" term, but there are problems with other operators.[5,13] Anyway, the region below 1 GeV is hardly accessible experimentally, because of the $b \to c$ background.

The conclusion at this point is that we can predict the photon spectrum for not too small and not too big energies, and moments of the photon spectrum from not too small energies to the endpoint. Making such predictions requires calculating matrix elements of various local operators among $\bar{B}$-meson states. Matrix elements of higher-dimensional operators are suppressed by higher powers of Λ/m_B. Therefore, we can write a double expansion for the "77" term, i.e. we can write

$$\sum_{X_s} BR[\bar{B} \to X_s\gamma]_{E_\gamma > 1 \text{ GeV}} = \left[a_{00} + a_{02} \left(\frac{\Lambda}{m_B} \right)^2 + ... \right] \\ + \frac{\alpha_s(m_b)}{\pi} \left[a_{10} + a_{12} \left(\frac{\Lambda}{m_B} \right)^2 + ... \right] + \mathcal{O}\left[\left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \right] \\ + [\text{ Contributions other than the "77" term}]. \tag{8}$$

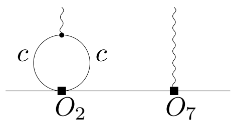
Here, the two terms not suppressed by Λ/m_B are simply those already found in the perturbative calculation of the b-quark decay. The term proportional to a_{02} turns out to give only an around -3% contribution.[12] The remaining terms in the first two lines of Eq. (8) have stronger suppression factors, which makes them negligible.

However, we need to ask whether a similar expansion can be written for the third line of Eq. (8), i.e. for the contributions other than the "77" term. The answer to this question is *no*. These remaining contributions contain, for instance, the huge effect from intermediate ψ that has been mentioned in the beginning of this talk. The intermediate ψ becomes important either due to non-perturbative effects or because of big contributions at high orders of perturbation theory that need to be resummed. Most probably, both mechanisms are at work.

The intermediate ψ contribution can be just subtracted from both the experimental data and the theoretical predictions, using the narrow peak approximation as in Eq. (1). Other narrow $c\bar{c}$ resonances hardly ever decay radiatively to charmless states, so similar contributions from them are negligible.

Suppose we subtract the intermediate ψ contribution. Does the sum of the remaining contributions take the form of a power series as in the first two lines of Eq. (8), with the perturbatively calculable leading term?

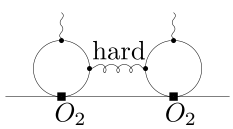
It does not.[5,15] However, it is hard to identify any obvious source of a big non-perturbative effect in it. Operators containing no charm quark are suppressed by their small Wilson coefficients. As far as the operators containing the charm quark are concerned, we know that their contribution at the leading order in α_s can be expressed as a power series

$\langle \bar{B}|$ 

$$|\bar{B}\rangle = (\text{perturbative } 0) + \frac{\Lambda^2}{m_c^2} \sum_{n=0}^{\infty} b_n \left(\frac{m_b \Lambda}{m_c^2} \right)^n, \quad (9)$$

which can be truncated to the leading $n = 0$ term, because the coefficients b_n decrease fast with n. The calculable[5,14] $n = 0$ term makes $BR[\bar{B} \to X_s \gamma]$ increase by around 3%.

However, an analysis of non-perturbative effects in the matrix elements of O_1 and O_2 at $\mathcal{O}(\alpha_s)$ is missing. For instance,

$\langle \bar{B}|$ 

$$|\bar{B}\rangle = A_{\text{one-loop}} + B_\psi + C_{\text{unknown}}, \quad (10)$$

where $A_{\text{one-loop}}$ stands for the very small (less than 1% in BR) perturbative contribution from the gluon bremsstrahlung at one loop, B_ψ is a part of the (huge) intermediate ψ contribution, and C_{unknown} denotes the remaining non-perturbative terms. Those remaining terms would not be numerically important if they were either suppressed by $\Lambda/m_{c,b}$, or small for other reasons, or could be absorbed into the intermediate ψ contribution. Unfortunately, I am not aware of any sufficiently precise argument that any of these three possibilities is realized.

In the following, I shall assume that one of these three possibilities is realized. In such a case, the hadronic decay rate is indeed well-approximated by the partonic decay rate, up to small non-perturbative corrections

$$\frac{\Gamma[\bar{B} \to X_s \gamma]^{\text{subtracted } \psi}_{E_\gamma > E_{\text{cut}}}}{\Gamma[\bar{B} \to X_c e \bar{\nu}_e]} \simeq \frac{\Gamma[b \to X_s \gamma]^{\text{perturbative NLO}}_{E_\gamma > E_{\text{cut}}}}{\Gamma[b \to X_c e \bar{\nu}_e]^{\text{perturbative NLO}}} \times$$
$$\times \left[1 + (\mathcal{O}(\Lambda^2/m_b^2) \simeq 1\%) + (\mathcal{O}(\Lambda^2/m_c^2) \simeq 3\%) \right]. \quad (11)$$

The normalization to the semileptonic rate has been used here to cancel uncertainties due to m_b^5, CKM-angles and some of the non-perturbative corrections. One has to remember that Eq. (11) becomes a bad approximation for $E_\gamma^{\text{cut}} << 1$ GeV, and that non-perturbative corrections grow dramatically[17] when $E_\gamma^{\text{cut}} > 2$ GeV.

For $E_{\rm cut} = 1$ GeV, Eq. (11) gives

$$BR[\bar{B} \to X_s \gamma]^{\rm subtracted\ \psi}_{E_\gamma > E_{\rm cut}} = (3.29 \pm 0.33) \times 10^{-4}, \tag{12}$$

where the dominant uncertainties originate from the uncalculated $\mathcal{O}(\alpha_s^2)$ effects and from the ratio m_c/m_b in the semileptonic decay (around 7% each).

Unfortunately, $E_{\rm cut} = 1$ GeV is not accessible experimentally at present. We need some prediction for the photon spectrum. The solid line in Fig. 5 describes the photon spectrum[15] in the region where the theoretical HQET prediction is solid. For larger energies, the less solid the line becomes, the less solid the prediction is. In the peak region, it is simply an "artist view" of how the spectrum could look like. However, its normalization is fixed by Eq. (12), and the size of the visible $\bar{K}^*(892)$ peak is adjusted to the value measured by CLEO.[16]

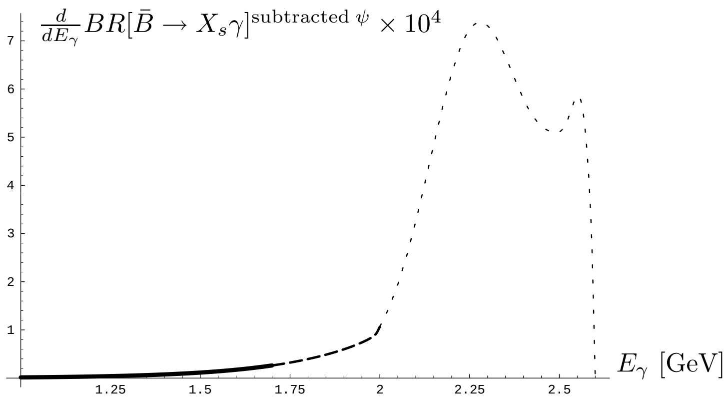


Figure 5. An "artist view" of $\frac{d}{dE_\gamma} BR[\bar{B} \to X_s \gamma]^{\rm subtracted\ \psi}$.

If we knew the shape function of the $\bar{B}$ meson exactly, we could make a solid prediction for the photon spectrum in the peak region, too.[17] Unfortunately, only models for the shape function are available at present. Therefore, an optimal scenario for a comparison between theory and experiment would be measuring the photon spectrum above ~ 2 GeV without relying on any theoretical prediction for its shape, and then extrapolating in a simple manner to the predicted spectrum below ~ 2 GeV. The measured spectrum above 2 GeV would provide[18] important information for extracting V_{ub} from $\bar{B} \to X_u l \nu$.

The decay $\bar{B} \to X_d\ \gamma$ is theoretically more difficult than $\bar{B} \to X_s\ \gamma$, because diagrams with up-quark loops are no longer CKM-suppressed with respect to the remaining ones. Thus, the theoretical accuracy is at best $\pm 30\%$, even for fixed Wolfenstein parameters ρ and η.

When the non-perturbative effects in up-quark loops are assumed to be small, one obtains[19]

$$\frac{1}{2}\{BR[\bar{B} \to X_d\ \gamma] + BR[B \to \bar{X}_d\ \gamma]\} \simeq 2.43[(1-\bar{\rho})^2 + \bar{\eta}^2 - 0.35(1-\bar{\rho}) \\ +0.07] \times 10^{-5} = 1.61 \times 10^{-5} \quad (\text{for } \bar{\rho} = 0.11 \ \text{ and } \bar{\eta} = 0.32), \tag{13}$$

while the direct CP-asymmetries range from 7% to 35%. Estimates for the exclusive channels are: $BR[B \to \rho^{\pm}\gamma] \in [1,4] \times 10^{-6}$ and $BR[B \to (\rho^0, \omega)\gamma] \in [0.5, 2] \times 10^{-6}$.

The present experimental results for $\bar{B} \to X_s\gamma$ already place severe constraints on extensions of the SM, like 2HDM, MSSM, LR-models etc. Theoretical predictions for exotic contributions have been recently calculated at NLO in many extensions of the SM.[20] These NLO effects are important only when the exotic effects are large, but tend to cancel among each other and/or the SM contribution, so that the present experimental constraints are satisfied.

The CP-asymmetry in $B \to K^*\gamma$ is very small in the SM, but could be significantly enhanced[11] in such extensions of the SM, in which the flavour-changing interactions of the right-handed s-quark are not suppressed by m_s/M_W, e.g. in the left-right symmetric models. The present CLEO bound[16] on the CP-asymmetry places important constraints on such models. Interesting information on physics beyond the SM can be obtained from the CP-asymmetry in the inclusive $\bar{B} \to X_s\gamma$ mode, too.[21]

To conclude:

- The present theoretical prediction for $BR[\bar{B} \to X_s\gamma]$ in the SM agrees very well with the measurements of CLEO and ALEPH. However, an analysis of non-perturbative effects at order $\mathcal{O}(\alpha_s)$ is necessary in order to make sure that the theoretical uncertainties are indeed around 10%.
- Future measurements of $BR[\bar{B} \to X_s\gamma]$ should rely as little as possible on theoretical predictions for the precise shape of the photon spectrum above ~ 2 GeV.

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